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SYNERGETIC THEORY OF ULTRA-THIN LUBRICANT FILM MELTING

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We represent a melting of ultra-thin lubricant film at friction between atomically flat surfaces as a result of action of spontaneously appearing elastic field of stress shear component that is caused by the external supercritical heating. The kinetics of this solid-liquid transition is described by the Maxwell-type and Voigt-Kelvin equations for viscoelastic matter as well as by relaxation equation for temperature. We show that these equations coincide formally with the synergetic Lorenz system, where the stress acts as the order parameter, the conjugate field is reduced to the elastic shear strain, and the temperature is as the control parameter. Using the adiabatic approximation we find the steady-state values of these quantities. Taking into account the deformational defect of the shear modulus we show that lubricant melting is realized according to mechanism of the first-order transition. The critical temperature of friction surfaces increases with growth of the characteristic value of shear viscosity and decreases with growth of the shear modulus value linearly.

Key words: friction, elastic shear stress and strain, Lorenz system, shear viscosity and modulus.

The interest to the problem of sliding friction is caused by its applied engineering importance [1]. One of the main goals of studies in this field is to define the conditions for low friction. At this direction the experiments were carried out with atomically flat mica surfaces separated by ultra-thin layer of liquid lubricant that manifested a solid-type behaviour at defined experimental conditions. Particularly, the stick-slip (or interrupted) motion has been observed at critical yield stress inherent in solid friction [2]. This effect occurs when lubricant film consists of several molecular layers and is explained as a confinement-induced freezing. The consequent melting takes place when shear stress is above some critical value due to "shear-induced melting" effect. Based on numerical methods work [3] maintains that liquid molecular ordering is realized owing to walls confinement. Studies stated in [4] - [6] were initiated for quantitative description of experimental results, in particular, in Ref. [6] the lubricant behaviour was described using the order parameter determining the melting degree. On the basis of viscoelastic matter approximation and Ginzburg-Landau equation, where order parameter defines the shear melting and freezing, the observed phenomenology of ultra-thin fluid film in the

process of friction is successfully described in [7]. Here the phase diagram is calculated defining the domains of sliding, stick-slip, and dry friction in the plane temperature – film thickness.

The starting point of our approach is the synergetic concept of phase transition [8,9] that is the generalization of phenomenological Landau theory. According to the latter, phase transition is controlled by an order parameter, over which value ϕ a free energy F is developed in a power expansion [10]. The equilibrium value of ϕ is determined by the condition

$$\partial F / \partial \phi = h, \quad (1)$$

where h is the external field and F is the free energy at $h = 0$. The relaxation process of transition to equilibrium is described by equation [11]

$$\dot{\phi} = -\frac{1}{\eta} \left(\frac{\partial F}{\partial \phi} - h \right). \quad (2)$$

Here η is the kinetic coefficient, which can be considered as the generalized viscosity. If ϕ is close to its equilibrium value $\phi_0 = 0$ we can use the linear approximation $\partial F / \partial \phi \approx \phi / \chi$, where $\chi \equiv \partial \phi / \partial h = \left(\partial^2 F / \partial \phi^2 \right)^{-1}$ is the susceptibility. As a result, the relaxation equation (2) takes the linear form

$$\tau \dot{\phi} = -\phi + \chi h, \quad (3)$$

where

$$\tau = \chi \eta \quad (4)$$

is the relaxation time.

The Eqs. (3), (4) were used by Landau and Khalatnikov to study the anomalous ultrasound absorption in the vicinity of phase transition. They held the fact that here susceptibility $\chi \rightarrow \infty$ and supposed that viscosity η is practically independent on temperature T . In their theory the anomalous large magnitude of τ is responsible to freezing process.

For viscoelastic matter the shear modulus G plays a role of the inverse susceptibility and expression (4) assumes the form

$$\tau = \eta / G. \quad (5)$$

In the cases of viscoelastic and displacement-type phase transitions (for example, of martensite-type) the modulus G goes to zero in the vicinity of transition point and relaxation time (5) diverges [12, 13]. There are works, see for example [14], using the fact that generalized susceptibility is practically independent on temperature but viscosity strongly increases with temperature at glass transition. Let us note that anomalous large value of equilibrium coefficient (susceptibility) is typical for phase transition and anomalous large value of kinetic coefficient is inherent in glass transition.

We are aiming to take into account, along the line [7], that the solid-liquid transition of ultra-thin lubricant film occurs due to both thermodynamic and shear melting. We obtain the unified analytical description of these processes as a result of the self-organization caused by the positive feedback of shear stress and temperature on shear strain - on the one hand, as well as the negative feedback of shear stress and strain on temperature - on the other one. Our approach is based on the assumption that relaxation time $\tau \rightarrow \infty$ because the shear viscosity diverges at the point of transition. The feature of

using the synergetic approach is that it allows us to obtain the synergetic potential, which is analog of free energy, from some simple equations.

The main supposition of our approach is that relaxation equation of the shear component σ of elastic stress tensor has the form similar to the Landau-Khalatnikov equation (3):

$$\tau_\sigma \dot{\sigma} = -\sigma + G\varepsilon. \quad (6)$$

Here ε is the corresponding component of strain, the first term on the right-hand side describes the Debye relaxation during time $\tau_\sigma \equiv \eta_\sigma / G$ determined by values of effective viscosity η_σ and shear modulus G . In the stationary case $\dot{\sigma} = 0$ the kinetic equation (6) is transformed into the Hooke law

$$\sigma = G\varepsilon. \quad (7)$$

Relaxation behavior of viscoelastic matter is described also by the Voigt-Kelvin equation [15]

$$\dot{\varepsilon} = -\varepsilon/\tau_\varepsilon + \sigma/\eta, \quad (8)$$

where τ_ε is the relaxation time of matter strain and η is the shear viscosity coefficient. The second term on the right-hand side describes the flow of a viscous liquid caused by the shear components of the elastic stress. In the stationary case $\dot{\varepsilon} = 0$ we obtain Hooke type expression $\sigma = G_\varepsilon \varepsilon$. It is worth to note that effective values of viscosity $\eta_\sigma \equiv \tau_\sigma G$ and modulus $G_\varepsilon \equiv \eta/\tau_\varepsilon$ do not coincide with the real values η and G . The formal reason for this difference is that the Maxwell-type equation (6) does not reduce to the Voigt-Kelvin equation (8) [16]. It is very important for our consideration that the values G_ε , G , η_σ depend on temperature T very weak, while the real viscosity η diverges, if the temperature decreases to point T_c [14]. Further we will use the simplest approximate temperature dependencies $G_\varepsilon(T)$, $G(T)$, $\eta_\sigma(T) = const$,

$$\eta = \frac{\eta_0}{T/T_c - 1}, \quad (9)$$

where $\eta_0 \equiv \eta(T = 2T_c)$ is the typical value of viscosity.

According to synergetic ideology [8, 9] for completing of the equations system (6), (8), that contain the order parameter σ , the conjugate field ε , and the control parameter T , it is necessary to add a kinetic equation for the temperature. This equation can be obtained using the basic equations of elasticity theory stated in [16]. Thus, we should start from the equation connecting the time derivatives of entropy S and internal energy U with equilibrium stress σ_0

$$T \frac{dS}{dt} = \frac{dU}{dt} - \sigma_0 \frac{d\varepsilon}{dt} \quad (10)$$

(in equilibrium the heat variation is $dQ = TdS$). In non-equilibrium case of non-uniform medium heating this equation has the form

$$-\text{div } \mathbf{q} = \frac{dU}{dt} - \sigma \frac{d\varepsilon}{dt}. \quad (11)$$

Here the heat current is given by the Onsager equation

$$\mathbf{q} = -\kappa \nabla T, \quad (12)$$

where κ is the heat conductivity constant, and the total stress $\sigma = \sigma_0 + \sigma'$ includes the viscous part σ' . Deducting Eq. (11) from Eq. (10), taking into account the equality

$$\frac{dS}{dt} = \frac{\partial S}{\partial U} \left(\frac{\partial U}{\partial T} \right)_\varepsilon \frac{dT}{dt} + \frac{\partial S}{\partial U} \frac{\partial U}{\partial \varepsilon} \frac{d\varepsilon}{dt} + \left(\frac{\partial S}{\partial \varepsilon} \right)_U \frac{d\varepsilon}{dt} = \frac{\rho c_v}{T} \frac{dT}{dt} + \frac{1}{T} \frac{\partial U}{\partial \varepsilon} \frac{d\varepsilon}{dt} - \frac{\sigma_0}{T} \frac{d\varepsilon}{dt},$$

and supposing that a layer of lubricant and atomically flat mica surfaces have different temperatures T and T_e , respectively, we obtain

$$\rho c_v \dot{T} = \frac{\kappa}{l} (T_e - T) + \sigma' \dot{\varepsilon} + T \frac{\partial \sigma_0}{\partial T} \dot{\varepsilon}. \quad (13)$$

Here the equalities $(\kappa/l)(T_e - T) \approx -\text{div } \mathbf{q}$ and $\partial U / \partial \varepsilon = \sigma_0 - T \partial \sigma_0 / \partial T$ are used, l is the scale of heat conductivity, ρ is the mass density, c_v is the specific heat capacity. The first term on the right-hand side of Eq. (13) describes the heat transfer from the layer of lubricant to friction surfaces. The second term takes into account the effect of the dissipative heating of a viscous liquid flowing under the action of the stress [17]. The third term represents the heat source that is conditioned by the reversible mechanic-and-caloric effect for which in linear approximation $T(\partial \sigma_0 / \partial T) \dot{\varepsilon} \approx \sigma_0 \dot{\varepsilon}$. As a result, the equation of heat conductivity can be written in the form

$$\rho c_v \dot{T} = \frac{\kappa}{l} (T_e - T) + \sigma \dot{\varepsilon}. \quad (14)$$

It is convenient to introduce the following measure units

$$\sigma_s = (\rho c_v \eta_0 T_c / \tau_T)^{1/2}, \quad \varepsilon_s = 1, \quad T_c \quad (15)$$

for variables σ , ε , T , respectively ($\tau_T \equiv \rho l c_v / \kappa$ is the time of heat conductivity). Then, substituting in (14) the expression for the $\dot{\varepsilon}$ from Eq. (8), the basic equations (6), (8), (14) take the form:

$$\begin{cases} \tau_\sigma \dot{\sigma} = -\sigma + (g/\theta)\varepsilon, & (16) \\ \tau_\varepsilon \dot{\varepsilon} = -\varepsilon + \theta(T-1)\sigma, & (17) \\ \tau_T \dot{T} = (T_e - T) - \sigma\varepsilon/\theta + \sigma^2, & (18) \end{cases}$$

where we introduce the constants

$$g = \frac{G}{G_0}, \quad G_0 \equiv \frac{\eta_0}{\tau_\varepsilon}; \quad \theta = \frac{\sigma_s}{G_0} \equiv \left(\frac{\tau_\varepsilon}{\tau_T} \right)^{1/2} \left(\frac{\rho c_v T_c \tau_\varepsilon}{\eta_0} \right)^{1/2}. \quad (19)$$

It is easy to see that the equations (16) - (18) have the form similar to the Lorenz system [8, 9].

For the qualitative analysis of this system let us use the adiabatic approximation, when the characteristic time scales are submitted to the following inequalities:

$$\tau_\varepsilon \ll \tau_\sigma, \quad \tau_T \ll \tau_\sigma. \quad (20)$$

They mean that in the course of medium evolution the strain $\varepsilon(t)$ and the temperature $T(t)$ follow the change of the stress $\sigma(t)$. The first of these conditions compares the macroscopic time τ_σ and microscopic Debye time $\tau_\varepsilon \sim 10^{-12}$ s, so that is satisfied always. Using the definitions of the thermometric conductivity $\chi \equiv \kappa/c_v$, the effective kinematic viscosity $\nu_\sigma \equiv \eta_\sigma/\rho$, and the sound velocity $c \equiv (G/\rho)^{1/2}$, it is conveniently to give to the second condition (20) the form

$$l \ll L, \quad (21)$$

according to that the characteristic length of heat conductivity has not to exceed the value

$$L = \frac{\chi v_{\sigma}}{\rho c^2}. \quad (22)$$

Then, we can set the left-hand sides of Eqs. (17), (18) to be equal to zero. As a result, the dependencies of the strain ε and the temperature T on the stress σ read:

$$\theta^{-1} \varepsilon = \sigma - (2 - T_e) \frac{\sigma}{1 + \sigma^2}, \quad (23)$$

$$T = T_e + (2 - T_e) \frac{\sigma^2}{1 + \sigma^2}. \quad (24)$$

According to Eq. (23) at $T_e < 1$ the ε vs σ dependence acquires a minimum at $\sigma = \sigma_+$, defined by equality

$$\sigma_{\mp} = \frac{1}{2} \left[(T_e - 4) \mp \sqrt{(T_e - 2)(T_e - 10)} \right]^{1/2}, \quad (25)$$

and then increases monotonically¹. When $T_e > 1$ the strain ε increases with stress σ linearly at Hook domain $\sigma \ll 1$. For values of T_e in the (1, 10) interval the ε vs σ dependence has a monotonically increasing shape with its minimum at point $\sigma = 0$. At $T_e = 10$ a plateau appears, which for $T_e > 10$ is transformed into a maximum and a minimum corresponding to the stresses σ_- and σ_+ , respectively. The temperature T at $T_e < 2$ increases with stress σ from the temperature T_e at $\sigma = 0$ to the horizontal asymptote $T = 2$, and at $T_e > 2$ control parameter decreases from the maximum value T_e at $\sigma = 0$ to the same asymptote $T = 2$. Obviously, this decrease is caused by the negative feedback of the stress σ and the strain ε on the temperature T in Eq. (18) that is the reflection of Le Chatelier principle for examined problem. On the other hand, the positive feedback of the stress σ and the temperature T on the strain ε in Eq. (17) is the reason for melting that leads to the growth of ε due to solid phase instability. However, in accordance with Eq.(18), the latter results in decrease of T as a consequence of self-organization process.

The pointed out positive feedback of σ and T on ε implies that the transition of lubricant from solid to fluid state is induced both by heating and under influence of stress generated by solid surfaces at friction. This agrees with examination of solid state instability within the framework of shear and dynamic disorder-driven melting representation in absence of thermal fluctuations [7].

According to Eq. (13) the negative feedback in Eq. (18) is always realized in course of relaxation process to the stationary state during that $\dot{\varepsilon} < 0$. As is shown in [9] this condition is satisfied in adiabatic case (20). The pointed out feedback may be conditioned also by the mechanic-and-caloric effect provided that $\partial \sigma_0 / \partial T < 0$.

The insertion of Eq. (23) into Eq. (16) gives the Landau-Khalatnikov-type equation

$$\tau_{\sigma} \dot{\sigma} = -\partial V / \partial \sigma, \quad (26)$$

where the synergetic potential reads:

$$V = \frac{1}{2} (1 - g) \sigma^2 + g \left(1 - \frac{T_e}{2} \right) \ln(1 + \sigma^2) \quad (27)$$

¹ The decrease domain of $\varepsilon(\sigma)$ function has no physical meaning.

At steady state the condition $\dot{\sigma} = 0$ is realized and potential (27) assumes a minimum. If the temperature T_e is smaller than the critical value

$$T_c = 1 + g^{-1}; \quad g = G/G_0 < 1, \quad G_0 \equiv \eta_0 / \tau_\varepsilon, \quad (28)$$

this minimum corresponds to the stress $\sigma = 0$, so that the melting can not take place and the solid state is realized. At the opposite case $T_e > T_c$ the stationary shear stress has the non-zero value

$$\sigma_0 = \left(\frac{gT_e - (g+1)}{1-g} \right)^{1/2} \quad (29)$$

increasing with T_e growth according to the root law. This causes the melting of film and its transition into fluid state. In accordance with Eqs. (23), (24) the corresponding stationary values of melting strain and temperature are as follows:

$$\varepsilon_0 = (\sigma_s / G)\sigma_0, \quad T_0 = 1 + g^{-1}. \quad (30)$$

It is interesting that, on the one hand, the melting temperature T_0 coincides with critical value (28) and, on the other hand, its value differs from the temperature T_e . The latter circumstance takes place due to that at steady state the first equation (30) is realized instead of connection $\sigma_0 = \varepsilon_0 / \theta$. Since T_c is the minimum value of temperature at which a solid-liquid transition begins, the above means that the negative feedback of the elastic stress σ and the strain ε on the temperature T (see third term on the right-hand side of Eq.(18)) reduces the film's temperature so much that only in the limit does it ensure the self-organization process. At steady state the melting value of shear viscosity coefficient is

$$\eta_m = \eta_0 g. \quad (31)$$

There are two opposite situation depending on parameter $g = G/G_0$ value. At the case $g \gg 1$, that is realized for small value of the viscosity coefficient η_0 , Eqs. (28)-(30) take the form:

$$\sigma_0 = (1 - T_e)^{1/2}, \quad T_0 = T_c = 1. \quad (32)$$

Such situation corresponds to the limit of strongly viscous liquid. At the opposite case $g \ll 1$ (large viscosity coefficient η_0) we have instead of Eq. (32) the solid (fragile) limit

$$\sigma_0 = (gT_e - 1)^{1/2}, \quad T_0 = T_c = g^{-1} \equiv \eta_0 / \tau_\varepsilon G. \quad (33)$$

The Maxwell equation (6) assumes the use of the idealized Genki model. For the dependence $\sigma(\varepsilon)$ of the stress on the strain this model is represented by the Hooke expression $\sigma = G\varepsilon$ at $\varepsilon < \varepsilon_m$ and the constant $\sigma_m = G\varepsilon_m$ at $\varepsilon \geq \varepsilon_m$ (σ_m , ε_m are the maximal stress and strain, $\sigma > \sigma_m$ leads to viscous flow with the deformation rate $\dot{\varepsilon} = (\sigma - \sigma_m) / \eta$). Actually, the dependence $\sigma(\varepsilon)$ curve has two regions: first one, Hookean, has the large slope fixed by the shear modulus G , and it is followed by the more gently sloping section of the plastic deformation whose tilt is defined by the hardening factor $\Theta < G$. Obviously the above picture means that the shear modulus, introduced (in terms of the relaxation time τ_σ) in Eq. (6), depends on the stress value. We use the simplest approximation

$$G(\sigma) = \Theta + \frac{G - \Theta}{1 + \sigma / \sigma_p}, \quad (34)$$

which describes the above represented transition of the elastic deformation mode to the plastic one. It takes place at characteristic value of the stress σ_p which does not exceed the value σ_s (in other case the plastic mode is not manifested). As a result, the relaxation time τ_σ obtains the dependence on the stress value:

$$\frac{1}{\tau_\sigma(\sigma)} = \frac{1}{\tau_p} \left(1 + \frac{\theta^{-1} - 1}{1 + \sigma/\sigma_p} \right), \quad (35)$$

where the relaxation time is introduced for the plastic flow mode (cf. Eq. (5))

$$\tau_p = \eta_\sigma / \Theta, \quad (36)$$

and the quantity

$$\theta = \Theta / G < 1 \quad (37)$$

is the parameter describing the ratio of the tilts for the deformation curve on the plastic and the Hookean domains. Note, that the expression type of Eq. (35) was offered, for the first time, by Haken [8] for the description of the rigid mode of the laser radiation. We used it [9] for the description of the first-order phase transition kinetics, however Eq.(35) had contained the square of the ratio σ/σ_p (so the V vs σ dependence had the even form in [9]). At description of the structural phase transitions of a liquid the third order invariants, breaking the specified parity, is present [10]. Therefore in approximation (35) we used the linear term σ/σ_p , instead of the square one $(\sigma/\sigma_p)^2$. It is visible, that the following below dependence (38) is not already even.

Within the adiabatic approximation (20) the system of the Lorenz equations (16)-(18), where instead of the τ_σ it is necessary to use dependence $\tau_\sigma(\sigma)$, is reduced as well as above, to the Landau-Khalatnikov equation (26) with τ_p instead of τ_σ . However in the synergetic potential (27) the factor $g = G/G_0$ is replaced by $g_\Theta = G^2/G_0\Theta < 1$, that is formally supposed to be no dependent on σ , and the odd term appears proportional to $\theta^{-1} - 1$:

$$V = \frac{1}{2}(1 - g_\Theta)\sigma^2 + g_\Theta \left(1 - \frac{T_e}{2} \right) \ln(1 + \sigma^2) + \alpha^2 (\theta^{-1} - 1) \left(\frac{\sigma}{\alpha} - \ln \left| 1 + \frac{\sigma}{\alpha} \right| \right). \quad (38)$$

Here the constant $\alpha \equiv \sigma_p/\sigma_s$ is introduced. At small value of temperature T_e dependence (38) has a monotonically increasing shape with its minimum at point $\sigma = 0$ corresponding to steady state of a solid. As it is seen from Fig. 1 at value

$$T_c^0 = 1 + (g_\Theta^{-1} - 1) \left(2\sqrt{p/3} \operatorname{ctg}(2\delta) + r/3 \right) \quad (39)$$

$$\operatorname{tg} \delta = \operatorname{tg}^{1/3}(\beta/2), \quad (|\delta| \leq \pi/4), \quad \operatorname{tg} \beta = \frac{2}{q} \sqrt{\left(\frac{p}{3}\right)^3}, \quad (|\beta| \leq \pi/2),$$

$$p = s - \frac{r^2}{3}, \quad q = \frac{2r^3}{3^3} - \frac{rs}{3} + t,$$

$$r = 3^3 \left(\frac{1}{3^2(1 - g_\Theta)} + \frac{\alpha^2}{2^2} - \frac{\gamma^2}{2^2 \cdot 3^3} - \frac{\gamma\alpha}{2 \cdot 3} \right), \quad \gamma = \frac{\alpha(\theta^{-1} - g_\Theta)}{1 - g_\Theta},$$

$$s = 3^3 \left[\frac{1}{1 - g_\Theta} \left(\frac{1}{3^2(1 - g_\Theta)} + \frac{\alpha^2\theta^{-1}}{2} - \frac{\gamma^2}{2 \cdot 3^3} - \frac{\gamma\alpha\theta^{-1}}{2 \cdot 3} - \frac{\gamma\alpha}{2 \cdot 3} \right) + \frac{\gamma^3\alpha}{3^3} \right],$$

$$t = 3^3 \left[\frac{1}{(1-g_\Theta)^2} \left(\frac{1}{3^3(1-g_\Theta)} + \frac{\alpha^2 \theta^{-2}}{2^2} - \frac{\gamma^2}{2^2 \cdot 3^3} - \frac{\gamma \alpha \theta^{-1}}{2 \cdot 3} \right) + \frac{\gamma^3 \alpha \theta^{-1}}{3^3(1-g_\Theta)} \right]$$

a plateau appears, which for $T_e > T_c^0$ is transformed into a minimum meeting the stress $\sigma_0 \neq 0$ and a maximum σ^m that separates minima corresponding to the values $\sigma = 0$ and $\sigma = \sigma_0$. With further growth of the temperature T_e the "ordered" phase minimum,

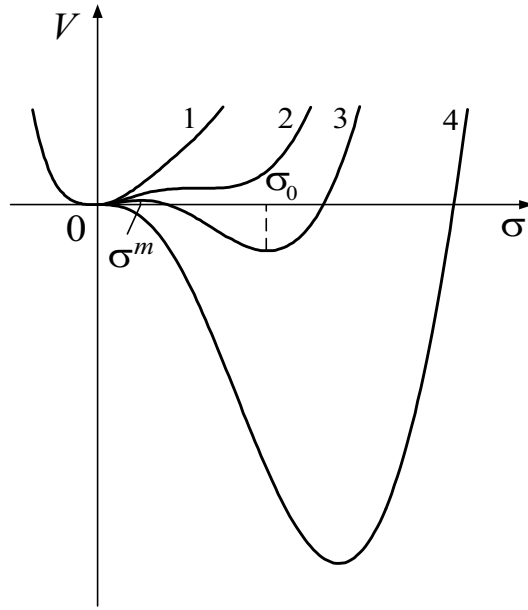


Fig. 1. The dependence of the synergetic potential on the elastic stress at various temperatures: (1) $T_e < T_c^0$; (2) $T_e = T_c^0$; (3) $T_c^0 < T_e < T_c$; (4) $T_e \geq T_c$

corresponding to a fluid state $\sigma = \sigma_0$, grows deeper, and the height of the interphase barrier decreases, vanishing at the critical value $T_c = 1 + g^{-1}$ (28). The steady-state values of the stress in a fluid state have the form (see fig 1 and 2):

$$\sigma_0 = 2\varpi \cos\left(\frac{\varphi}{3}\right) - \frac{\gamma}{3}, \quad (40)$$

$$\sigma^m = 2\varpi \cos\left(\frac{\varphi}{3} - \frac{2\pi}{3}\right) - \frac{\gamma}{3}; \quad (41)$$

$$\varpi \equiv (-\psi/3)^{1/2}, \quad \cos \varphi \equiv -\omega/2\varpi^3,$$

$$\psi = \zeta - \frac{\gamma^2}{3}, \quad \omega = \frac{2\gamma^3}{3^3} - \frac{\gamma\zeta}{3} + \xi,$$

$$\gamma = \frac{\alpha(\theta^{-1} - g_\Theta)}{1 - g_\Theta}, \quad \zeta = \frac{1 + g_\Theta(1 - T_e)}{1 - g_\Theta},$$

$$\xi = \frac{\alpha[\theta^{-1} + g_\Theta(1 - T_e)]}{1 - g_\Theta}.$$

At $T_e \geq T_c$ the dependence $V(\sigma)$ has the same character as in the absence of the modulus defect (see the curve 4 in fig. 1).

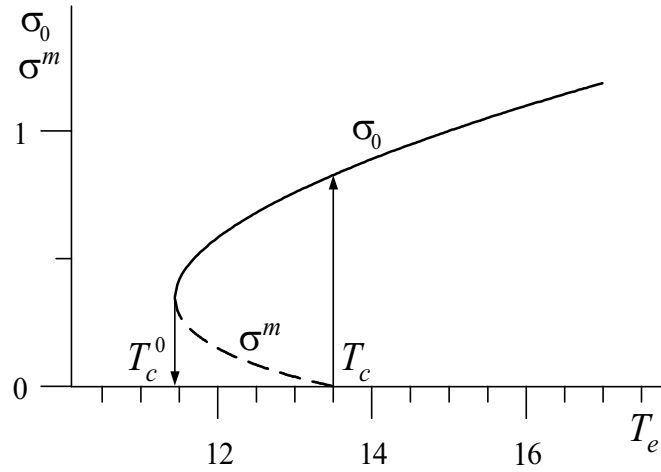


Fig. 2. The dependence of the steady-state values of the stress on the temperature T_e at $g_\Theta = 0.2$; $\theta = 0.4$; $\alpha = 0.5$ (the solid curve corresponds to the steady state value σ_0 , the dashed curve meets the unstable one σ^m)

The specified peculiarities corresponds to the positive stress values σ . On the negative half-axis at $T_e > T_c$ with the increase of $|\sigma|$ a very weak minimum of the V vs σ dependence is observed, which is followed by the infinite increase of the potential V at $\sigma = -\sigma_p$. Thus, the negative values of the elastic fields σ , ε are not realized practically.

The characteristic circumstance of our scheme is that energy barrier inherent in the synergetic first-order transition is displayed only at presence of the deformational defect of the modulus. Since latter takes place always, it follows that a studied solid-liquid transition represents synergetic first-order transition. The examined situation is much

more complex than usual phase transitions. Really, in the latter case the steady-state value of the system's temperature T_0 is reduced to the value T_e fixed by thermostat. In our case T_0 is reduced to the critical value T_c for the synergetic second-order transition, that has place in the absence of the modulus defect (see section 3). When we take into account the modulus defect the temperature

$$T_0 = T_e + (2 - T_e) \frac{\sigma_0^2}{1 + \sigma_0^2} \quad (42)$$

is realized whose value is defined by a minimum position of the dependence (38). According to Eqs. (40), (42) the quantity T_0 smoothly decreases from the value

$$T_m = T_c^0 + (2 - T_c^0) \frac{(\sigma_0^c)^2}{1 + (\sigma_0^c)^2}, \quad (43)$$

$$\sigma_0^c = \left[\frac{1}{3^{1/2}} \left(\frac{\gamma^2}{3} - \frac{1 + g_\Theta (1 - T_c^0)}{1 - g_\Theta} \right)^{1/2} - \frac{\gamma}{3} \right]^2$$

at $T_e = T_c^0$ to the $T_{c0} \equiv 1 + g_\Theta^{-1}$ at $T_e \rightarrow \infty$. Referring to fig. 3, the stationary temperature T_0 shows a linear increase from 0 to T_c with T_e being in the same

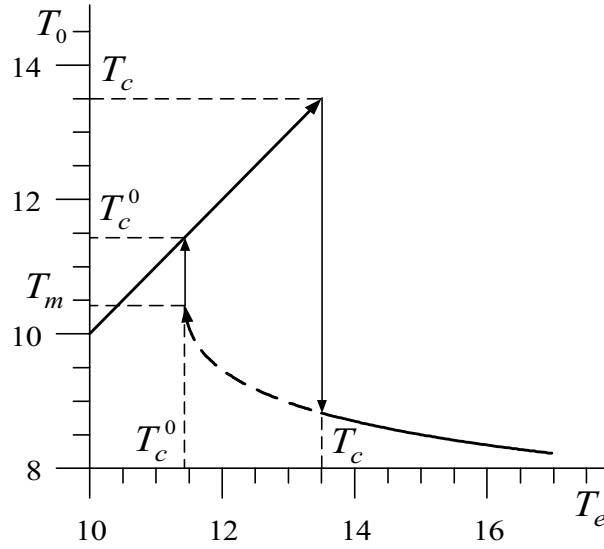


Fig. 3. The dependence of the steady-state value of the system temperature T_0 on the temperature T_e ($g_\Theta = 0.2$; $\theta = 0.4$; $\alpha = 0.5$)

interval and, after the jump down at $T_e = T_c$, magnitude T_0 smoothly decays. If the temperature T_e then decreases the stationary temperature T_0 grows. When the point T_c^0 (39) is reached T_0 undergoes the jump from T_m (43) up to T_c^0 . For $T_e < T_c^0$ again stationary temperature T_0 does not differ from T_e .

Since the stationary values of stress $\sigma_0 > 0$ are realized only at $T_e > 1 + g_\Theta^{-1}$, $g_\Theta < 1$ the range of values $T_c^0 > 2$ is important for consideration. In this interval the maximum system's temperature (43) is lower than the minimum temperature of friction surfaces (39), and as it is visible from Fig. 3, at $T_e > T_c^0$ the stationary temperature T_0 of the film is always lower than value T_e .

The above analysis is based on the assumption that a lubricant melting process is caused by the self-organization of the shear components of the stress and the strain elastic fields, on the one hand, and the lubricant temperature, on the other hand. Thus, the stress σ acts as the order parameter, the conjugate field is reduced to the elastic strain ε , and the temperature T is the control parameter. The initial reason for self-organization is the positive feedback of T and σ on ε (see Eq. (17)). According to Eqs. (8), (9) it is caused by the temperature dependence of the shear viscosity leading to its divergence. Accounting for the deformational defect of the shear modulus we obtain the expressions for temperatures corresponding to absolute instability of overcooled liquid T_c^0 (39) and stability limit of the solid state T_c (28). The real thermodynamic melting temperature is in the (T_c^0, T_c) interval and can be found from the equality condition of potentials of solid and liquid phases $V(0) = V(\sigma_0)$. It is seen from Eq. (28) that systems predisposed to melting have large shear modulus G and small characteristic value of shear viscosity η_0 .

The kinetics of a considered transition is determined by the Landau-Khalatnikov equation (26), where τ_σ is replaced by $\tau_p = \eta_\sigma / \Theta$ and the synergetic potential has the form (38) inherent in the first-order transition. In supercooled liquid with $\eta_\sigma = \infty$ the freezing of system can takes place ($\dot{\sigma} \rightarrow 0$) even in the non-stationary state $\partial V / \partial \sigma \neq 0$.

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СИНЕРГЕТИЧНА ТЕОРІЯ ПЛАВЛЕННЯ УЛЬТРАТОНКОЇ ПЛІВКИ МАСТИЛА

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Плавлення ультратонкої плівки мастила в процесі тертя між атомно плоскими поверхнями подають як результат дії пружного поля зсувної компоненти напружень, що спонтанно з'являються в разі зовнішнього надкритичного нагрівання. Кінетику переходу між твердим та рідким станами описують рівняння в'язкопружного середовища типу Максвелла і Фойгта-Кельвіна та релаксаційне рівняння для температури. З'ясовано, що ці рівняння формально збігаються з синергетичною системою Лоренца, де напруження відіграють роль параметра порядку, спряжене поле зводиться до пружної зсувної деформації і температура є провідним параметром. У рамках адіабатичного наближення знайдено стаціонарні значення цих величин. З урахуванням деформаційного дефекту модуля зсуву доведено, що плавлення мастила відбувається за механізмом переходу першого роду. Критична температура поверхонь тертя підвищується зі збільшенням характерного значення зсувної в'язкості і знижується зі збільшенням значення модуля зсуву за лінійним законом.

Ключові слова: тертя, пружна зсувна деформація, система Лоренца, модуль зсуву, в'язкість.

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